SECTION 6

THE WEIGHING OF GLIDERS

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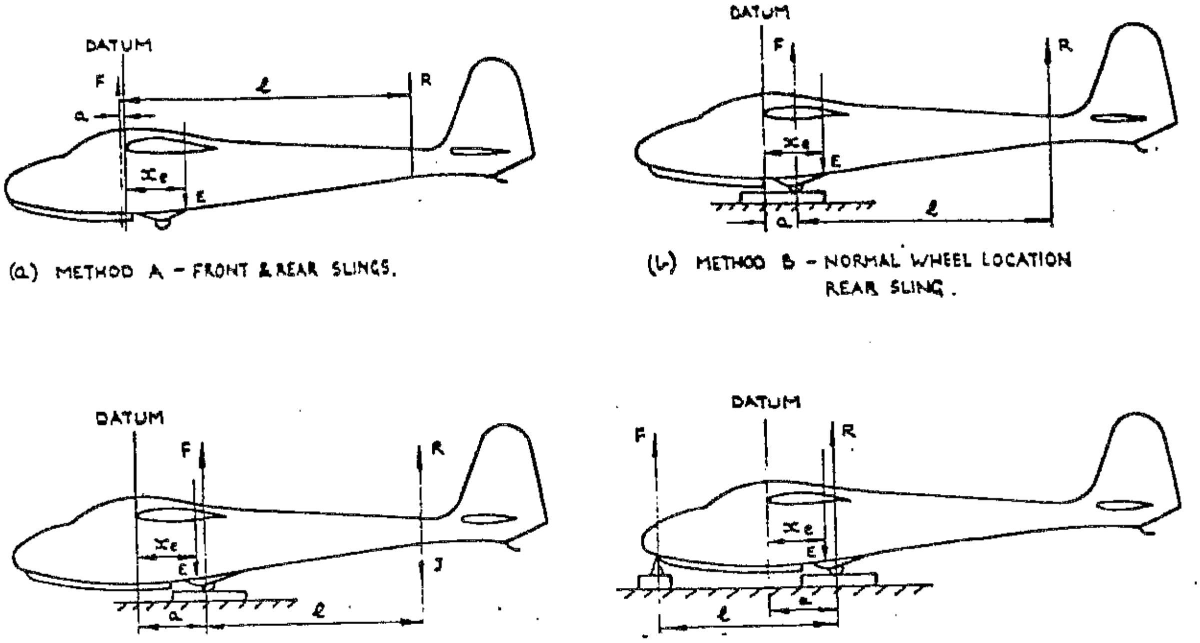


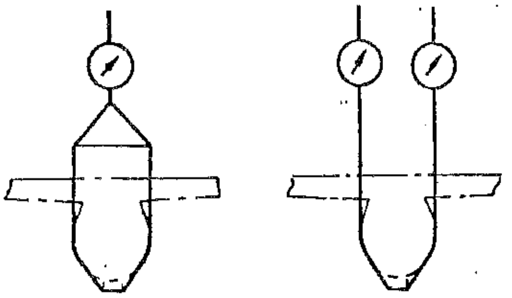
(a) METHOD A – FRONT & REAR SLINGS.

(b) METHOD B – NORMAL WHEEL LOCATION REAR SLING.

(c) METHOD B – AFT WHEEL LOCATION REAR SLING

(d) METHOD B.– AFT WHEEL LOCATION NOSE SUPPORT

FIG. 1. ALTERNATIVE SUPPORT METHODS.



(a) SINGLE BALANCE

(b) TWO BALANCES

FIG 2. FORWARD SLING – ALTERNATIVE METHODS.

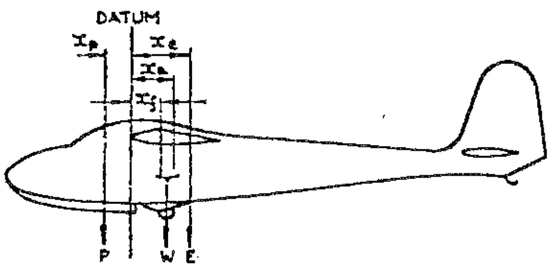


FIG. 3. LOADING DIAGRAM, SINGLE SEATER OR SIDE BY SIDE TWO SEATER

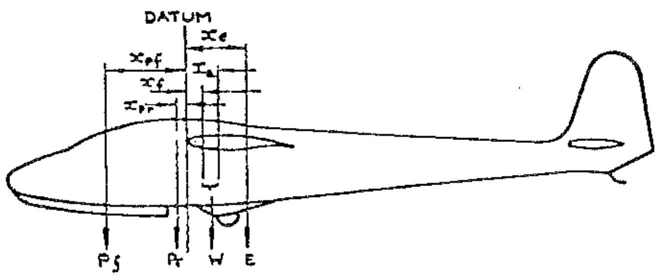


FIG. 4. LOADING DIAGRAM, TANDEM TWO SEATER

THE WEIGHING OF GLIDERS

1. Introduction

The paper has been written with the object of collecting together the major and, as far as possible, the minor points in both the theory and practice of glider weighing. It is hoped that it will prove helpful to those who have to carry out weighings, not only with respect to working out the answers, but in the way to tackle the actual job to obtain accurate and consistent results.

In paragraphs 5 and 6, the necessary mathematical formulae are derived. The various symbols are defined when first introduced, and a complete list is also given at the end of the paper, but before the appendices, for handy reference.

2. Object of Weighing

The object of weighing any glider is to determine the empty weight and C.G. position, and thence the loading limitations needed to ensure that the weight and C.G. of the loaded glider do not exceed the permissible limits. The latter will have been decided by the certificating authority in conjunction with the designer and the flight test group which does the flight tests.

The division of the total or all-up-weight into its two components empty weight and load is to a certain extent arbitrary, though clear enough. The empty weight is the weight of the airframe and all fixed instruments and equipment including oxygen and radio if fitted. The load is the pilot plus all loose equipment that is likely to vary between one flight and another, i.e. parachute, loose cushions, maps, food and drink. Ballast attached more-or-less permanently to the structure is usually counted in the empty weight, but any ballast which is regularly moved, removed or refitted according to whether the machine is to be flown by a light or heavy pilot is generally better regarded as load. Such things as barograph and camera may be considered as empty weight or load as desired—it does not matter which so long as they are not forgotten, and so long as the weight records in the aircraft log-book make it clear.

3. Importance of Correct Weight and C.G.

The importance of the all-up-weight limitation is obvious enough. The glider is designed to have a specified safety factor over the loads arising in manoeuvres of a specified severity. The loads in many of the design manoeuvres vary with weight, the variation being in some cases rather greater than that of direct proportion. Any increase in weight above the permitted maximum therefore eats into the safety factor in such manoeuvres.

The importance of the C.G. limits is that the C.G. position affects some of the flying characteristics. With the C.G. in front of the forward limit, the up elevator power may be insufficient for landing or for aerobatics, and if no trimmer is fitted an undue pull force on the stick may be required even at normal low speeds. Strength factors are also affected though usually only to a small degree. With the C.G. aft of the aft limit, the down elevator power may be inadequate for recovery from stalls or cable breaks, longitudinal stability will be reduced and may become negative, and stalling and spinning behaviour may be adversely affected. In this case strength is not usually critical. Put briefly, if the C.G. is too far forward the glider may be tiring to fly and difficult to land, and if the C.G. is too far aft it may be not only difficult to fly but dangerous.

4. Weighing Methods

Broadly, the weighing process consists of supporting the glider effectively at two points, one of which is in front of the C.G. and the other behind it, and noting the reactions at the supports and the distances of the latter from the datum to which C.G. positions are to be related. In some cases (see 4.4 below) use of a false datum may have to be made.

There are two main methods of support:

A — By front and rear hanging balances or scales—see Fig. 1(a).

B — By a platform scale under the main wheel and a rear (or in some cases a forward) support—see Figs. 1(b) to (d).

In method A, the front support may be a single spring balance with a bridle or a pair of balances with an open loop, as shown in Fig. 2(a) and 2(b) respectively. Arrangement (b) may be essential for two-seaters in order that the load may be coped with by the 400 lb. balances normally used by gliding clubs. Where the load is small enough, as with most single seaters, arrangement (a) is usually used and can yield satisfactory results, but arrangement (b) is recommended for all cases if an extra balance is available, as it confers lateral constraint and so prevents undue rolling about—wing-tip holding is eliminated.

In method B, the secondary support depends on the location of the main wheel. If the latter is ahead of the empty C.G., as usual, there can be a rear sling as in Method A—Fig. 1(b). If the wheel is aft of the empty C.G., two arrangements are possible. Either a rear sling can be used, with some jury ballast hung below it to give a positive load, i.e. to prevent the tail from rising—Fig. 1(c)—or a nose support, as in Fig. 1(d).

In Fig. 1, the secondary support is shown as a sling in cases (a), (b) and (c), and as a small platform scale in case (d). If desired, a platform scale could equally well be used in cases (a) and (b), say under the tail skid, or a sling in case (d). The calculations are not affected by which type of secondary support is chosen. Case (c) also could be adapted to use a platform scale for the secondary support, but would then have complications, and the proper thing to do if the wheel is aft and it is desired to use platform scales for both supports is to use case (d).

For platform-type secondary supports, bathroom scales have sometimes been used, but should be employed only if it is quite certain that they are in good condition and if they are calibrated by means of known weights at two levels bracketting the scale reading required.

4.1. *Method A—Front and Rear Slings*

For the highest accuracy in finding the C.G. position the front sling should be as close to the C.G. as possible, or more correctly the ratio of the distance between C.G. and rear sling to the distance between C.G. and front sling should be as large as possible. This is most nearly achieved when the front sling is placed adjacent to the wing leading edge, and this is what is recommended where it is practicable.

When the glider is suspended, the slings should be as nearly vertical as possible. Now if one sling is not vertical, the horizontal component of its load must be balanced by an equal and opposite horizontal component of the load in the other; the latter cannot therefore be vertical either. Hence, if the first sling is vertical, there is no horizontal component of load, and the other sling must be vertical also. Therefore it is only necessary to check the verticality of one sling, and since if the slings are inclined the one carrying the smaller load will be at the larger angle to the vertical, i.e. the one in which the error is easier to detect, this is the one to check. Normally this will be the rear sling. It is usually satisfactory to take a sight against a suitable girder or brick wall, but if there is any doubt a plumb line should be used.

The slings should, of course, be so placed round the fuselage as to eliminate or minimise any squeezing loads which might cause damage, i.e. should be at or at least close to a frame, and the front sling may have to be a little forward of the leading edge for this reason. If a single balance is used for the front sling a spreader bar above the fuselage will be required. Normally the canopy will be fitted, and the spreader must be suitably secured to the sling to prevent it from slipping down and causing damage. If the canopy is removed the spreader can rest on the longerons. If necessary a spreader can be used under the fuselage also, and since this one will be subjected to considerable bending loads it must be of ample strength.

Where the slings are of coarse rope or wire, as is often the case, suitable felt or rag padding should be used to protect the finish of the glider.

If the rear support is a sling it must be checked for verticality as described. If it is a bathroom scale, it must be packed as necessary to ensure that the platform is level; in this case the front support must be checked for verticality.

4.2. *Method B—Platform Scale*

With this method, there is less to say about the support details. The platform must, of course, be properly level and free in action. If a wheel brake is fitted to the glider, it must be off and free if the rear support provides fore and aft constraint, or an error in C.G. position may be introduced due to the torque; if there is no such constraint the point is immaterial because no torque can be reacted.

On a few aircraft, the empty C.G. is ahead of the wheel. In such cases, if method B is to be used, either suitable ballast will have to be secured under the rear sling to ensure a positive rear-support load, and allowed for in the calculations, or a nose support must be used.

If the glider has no wheel but only a skid, a suitable fulcrum bar must be placed between the skid and the platform, located at or adjacent to a strong-point and as near the C.G. as practicable. Unless a suitable load-spreader is used, the fulcrum should not be too sharp-edged, or the skid may be damaged, but should have a radius of say a quarter to half an inch. In this skid case, the rear support must be free horizontally to avoid the possibility of error due to friction torque.

4.3. *Precautions Applicable to both Methods*

Whichever method of support is used, there are certain additional precautions to be taken. First, the glider should be in the standard empty condition or as near to this as is possible, both in the interests of accuracy and to minimise subsequent calculation. Secondly, unless the day is absolutely still the hangar doors should be closed or more or less closed to eliminate wind loads. Thirdly, except when three balances are used a wing-tip holder will usually be required to prevent lateral rocking while the balances are being read, and it is essential that he holds the wing lightly between finger and thumb at the angle of bank which corresponds to zero load, which in general will not be 0 degrees since it will depend on the precise way in which the glider sits in the sling. Fourthly, the aircraft should be supported approximately at the correct attitude.

The last point needs some amplification. The correct attitude is normally specified by the manufacturers. For old gliders the information may not be available, and in such cases a sensible-looking attitude should be used; it is recommended that the wing lower surface, from spar to trailing-edge at or near the root, should be horizontal. Fortunately the vertical position of a glider C.G. is usually similar to the vertical position of the datum point, and this means that any error in attitude only affects the horizontal C.G. distance by a simple cosine factor and for such cases an accuracy of 2° to 3° is sufficient. Where the datum point is not the wing leading edge the above may not apply and the attitude should be set up as accurately as possible.

4.4. *Taking the Readings*

The glider having been supported by one of the methods described, and it having been ensured that it is free but steady (if it is not free, the readings may be false, and if it is not steady they will be variable), the actual measurements may be taken.

Balance readings should be noted to the nearest half pound—the weight will not be stable to such a limit or anywhere near it, but the C.G. is sensitive to quite small errors in the secondary support load.

Distances should be taken from the centres of the supports, and averages taken of the values measured on the port and starboard sides of the glider. Where platform scales are used, fine chalk marks must be made on the platform surface on both sides of the wheel, at the feet of plumb-bobs from the datum point (again on both sides) and from the rear or nose support as applicable. In this case, the averaging must be done by marking off the centres of the lines joining each pair of chalk marks after the glider has been moved out of the way. The three centres should be collinear, and the measurements made along the line joining them. The distance from the front sling or main wheel to the datum should be correct to 0·05 in. if possible, certainly to 0·1 in., but the distance between the two supports need only be accurate to 0·5 in. as a rule.

On many gliders the datum taken is the leading edge at the wing root. On these gliders the measurements can be made directly without difficulty; when using platform scales for the main support the plumb-lines, of cotton or thin string, are simply draped over the wing. Where some other datum is used, e.g. the spar, it is often more convenient to take measurements from the leading edge as usual, and then (removing the canopy if necessary) to make a separate measurement of the distance between the leading edge and the specified datum. The support distances may then be corrected appropriately.

After the glider has been moved, the zero readings of all balances or scales must be noted—they are subtracted from the previous readings to obtain the actual loads.

5. Calculation of Empty Weight and C.G. Position

This consists of two parts:

(a) The calculation in the condition as weighed.

(b) The correction to the standard empty condition.

As indicated above, the glider should preferably be weighed in the standard empty condition, i.e. with all the fixed instruments and equipment in position and all load items removed. In many cases this will be so, and in such cases step (b) is unnecessary.

It is recommended that the weight be expressed to the nearest pound and the C.G. to the nearest 0·1 in. or at the best 0·05 in. Any greater precision is quite unjustified either by experimental accuracy or, for wooden machines, the stability of the moisture content of the timber.

5.1. *Empty Condition as Weighed*

There are four different configurations as illustrated in Fig. 1, according to the precise method used for supporting the glider, but the symbols can be the same in all four, with one exception as noted.

Let F = front support load.
R = rear support load.
J = weight of jury ballast.
E = empty weight, as weighed.
a = distance from front sling to datum (in Method A), or from platform load to datum (in Method B including its variations).
l = distance from front support to rear support,
x_e = distance of empty C.G., as weighed, aft of datum.

Note that when using method B, a is taken from the platform support position irrespective of whether the platform is the front support (the usual case) or the rear support (as in the case of the empty C.G. being ahead of the wheel).

The formulae are however slightly different in the various configurations, and it is important not to mix them up.

Method A—Fig. 1(*a*)

By vertical forces $E=F+R$

By moments about F $E\,(a+x_e)=R\,l$

whence
$$x_e=\frac{R}{E}\,l-a=l\,\frac{R}{(F+R)}-a \qquad \ldots\ldots 1$$

Method B

Normal case with C.G. aft of wheel, or aft of skid support—Fig. 1(*b*)

By vertical forces $E=F+R$

By moments about F $E\,(x_e-a)=R\,l$

whence
$$x_e=\frac{R}{E}\,l+a=l\,\frac{R}{(F+R)}+a \qquad \ldots\ldots 2$$

C.G. ahead of wheel, jury ballast at rear sling—Fig. 1(c)
By vertical forces $E=F+R-J$
By moments about F $E(a-x_e)=(J-R)\,l$
whence

$$x_e=a-\frac{(J-R)}{E}l=a-\frac{(J-R)}{(F+J-R)}l \qquad \ldots\ldots 3$$

Alternatively J may be treated as an item of load and deducted by the method given in 5.2 below.
C.G. ahead of wheel, glider supported at nose—Fig. 1(d).
By vertical forces $E=F+R$
By moments about R $E(a-x_e)=F\,l$
whence

$$x_e=a-\frac{F}{E}l=a-\frac{F}{(F+R)}l \qquad \ldots\ldots 4$$

A set of worked examples is given in Appendix 1.

5.2. *Correction to Standard Empty Condition*

If the glider is weighed in the standard empty condition, the weight and C.G. position as weighed are also the corrected or standard values. If, however, some equipment is missing (e.g. one or more instruments removed for calibration) or some removable load is present (e.g. parachute left in) the correction is made as follows:

Each item or group of items is weighed, and the distance of its C.G. from the datum point is measured.

Let E^1 and x_e^1 be the weight and C.G. position as weighed, and
E and x_e be the corrected or standard values.
Let w_1, w_2 etc.=weights of missing or surplus items.
x_1, x_2 etc.=distance of missing or surplus items from datum point, $+ve$ if aft of it and $-ve$ if in front.

Then by vertical forces:

$$E=E^1+w_1 \text{ etc. for missing items}$$
$$-w_1 \text{ etc. for surplus items}$$

and by moments about the datum:

$$E\,x_e=E^1\,x_e^1+w_1\,x_1 \text{ etc. for missing items}$$
$$-w_1x_1 \text{ etc. for surplus items}$$

Then x_e is found by dividing E into the total moment.

A worked example is given in Appendix 2.

6. Loading Limitations

The calculations differ to some extent according to the number and layout of the seats in the glider. There are two cases:

(1) Single-seater and side-by-side two-seaters. These are taken together because there is only one cockpit C.G. and hence a single set of formulae is applicable to both.

If the seat and/or seat back is adjustable in position fore and aft, the cockpit C.G. is different for each position. For the highest accuracy account should be taken of this. To do so means making calculations for all the seat positions and placarding the several answers. In the ordinary case, however, it is sufficiently accurate to take one mean, or slightly aft position—say, the middle one of three equally spaced, or the third from the front of four equally spaced.

(2) Tandem two-seaters. These have two cockpit loads to be considered, and the two cockpit C.G's. are different. This introduces additional terms in the formulae and the limits for the load in one cockpit depend on the load in the other. Thus it is necessary to determine the maximum and minimum loads in one cockpit for a series of loads in the other. Normally the rear cockpit is

near the C.G. of the loaded aircraft, and for this reason it is convenient to take the rear cockpit load as the independent variable, i.e. to take arbitrary values for this and then calculate the corresponding front cockpit load limits.

The methods of calculating the maximum and minimum cockpit loads, and the derivation of the formulae, are as follows.

For convenience in use, all distances are defined so that they are represented by positive numbers in the normal case, and the equations written accordingly. Where a distance is measured in the reverse direction (e.g. cockpit C.G. aft of datum instead of in front) it must be written as a minus quantity.

6.1. *Single-Seaters and Side-by-Side Two-Seaters*

The minimum cockpit load is governed solely by the aft C.G. limit and hence only a moment sum is required.

Let P = cockpit load.

x_p = distance of cockpit C.G. ahead of datum.

x_f = forward C.G. limit, aft of datum.

x_a = aft C.G. limit, aft of datum.

and other symbols be as before.

By moments about the C.G. (aft limit):

$$P\,(x_p+x_a) = E\,(x_e-x_a)$$

$$\therefore \quad P = E\,\frac{(x_e-x_a)}{(x_p+x_a)} \qquad \ldots\ldots 5$$

The maximum cockpit load may be governed by the maximum all-up-weight or by the forward C.G. limit according to the case; a weight sum and a moment sum are therefore required. There are two methods of arranging these.

The usual method is first to determine the maximum from the all-up and empty weights, and then to check whether, with the resulting load, the C.G. is within limits.

Let W = total weight of loaded aircraft.

$\bar{x}$ = distance aft of datum of C.G. of loaded aircraft.

By weight difference: $P = W - E$ $\qquad \ldots\ldots 6$

Then by moments about the datum point:

$$W\bar{x} = E\,x_e - P\,x_p$$

$$\therefore \quad \bar{x} = \frac{1}{W}\,(E\,x_e - P\,x_p) \qquad \ldots\ldots 7$$

P is found from equation 6 and used in equation 7. If the $\bar{x}$ so obtained is within limits, i.e. between x_f and x_a, the result is valid and the above value of P is the answer. If $\bar{x}$ is not within limits, i.e. is less than x_f, if x_f is positive, or numerically more than x_f if x_f is negative, then the C.G. is the governing factor and not the all-up weight. In this event, it is necessary to proceed further in a similar manner to that for minimum load case.

By moments about the C.G. (forward limit):

$$P\,(x_p + x_f) = E\,(x_e-x_f)$$

$$\therefore \quad P = E\,\frac{(x_e-x_f)}{(x_p+x_f)} \qquad \ldots\ldots 8$$

The value of P found from equation 8 is then the required maximum load.

The second method is to use equation 6 as before, and then equation 8 instead of equation 7. This procedure gives two values of P, respectively one due to weight and the other due to C.G. The smaller of these is the required value. Just occasionally the two will be equal, which means that both factors are limiting simultaneously.

This second method is strongly recommended. For one thing, the answer is always obtained from two sums instead of sometimes two and sometimes three. Secondly the whole process (of finding both maximum and minimum loads) requires only one type of moment sum instead of two.

A complete worked example, including both methods for the maximum cockpit load, is given in Appendix 3.

6.2. *Tandem Two-seaters*

In this case, as indicated above, there are additional terms to be taken into account.

Let P_f = front cockpit load.
P_r = rear cockpit load.
x_{pf} = distance of front cockpit C.G. ahead of datum.
x_{pr} = distance of rear cockpit C.G. ahead of datum
and other symbols be as before.

The minimum front cockpit loads are, as before, governed solely by the aft C.G. limit.

By moments about the C.G. (aft limit):

$$P_f (x_{pf} + x_a) + P_r (x_{pr} + x_a) = E (x_e - x_a)$$

$$\therefore \quad P_f = \frac{E (x_e - x_a) - P_r (x_{pr} + x_a)}{(x_{pf} + x_a)} \qquad \ldots\ldots 9$$

P_f should be worked out for $P_r = 0$, 100, 120, 140, etc. lb. up to 240 lb. or until the resulting P_f becomes 100 lb. or so, whichever occurs first.

The maximum front cockpit loads may, as before, be governed by the maximum all-up-weight or by the forward C.G. limit; the governing factor may vary with rear cockpit load. Only the second, recommended, method is given in this case.

By weight difference: $P_f = W - E - P_r$ 10

By moments about the C.G. (forward limit):

$$P_f (x_{pf} + x_f) + P_r (x_{pr} + x_f) = E (x_e - x_f)$$

$$\therefore \quad P_f = \frac{E (x_e - x_f) - P_r (x_{pr} + x_f)}{(x_{pf} + x_f)} \qquad \ldots\ldots 11$$

P_f should be worked out, as above, for $P_r = 0$, 100, 120, 140 etc. lb. up to 240 lb. or, in this case, until the resulting P_f either rises to 240 lb. or so or falls to 100 lb. or so. Usually it will not vary greatly and will be between these two extreme values.

If most or all of the maximum values of P_f do come outside one of the above extremes, the fact may be an indication that the balance of the aircraft is unsatisfactory. If P_f is too high, the aircraft is likely to be tail heavy—this can be judged finally by the minimum values of P_f—and some nose ballast is required. If P_f is too low, the aircraft is nose heavy and some tail ballast is required. In either of these events, new values of E and x_e including the ballast must be determined—this can conveniently be done by the method of 5.2—and the maximum and minimum cockpit loads calculated afresh. The process must be repeated as required until satisfactory results are obtained.

Two worked examples are given in Appendix 3.

6.3. *Overall Limitation*

The maximum cockpit load as calculated may in some circumstances be very large—this frequently happens for the front cockpit of a tandem two-seater for low values of rear cockpit load. In such cases, an overall limitation of say 240 lb. should be taken, as greater values might result in excessive local structure loads. Greater values should only be quoted, if required, with the agreement of the manufacturers of the machine.

6.4. *Datum Point other than Leading Edge*

The equations given above are theoretically applicable for any datum point, minus signs being used when forward-defined distances are aft and *vice versa*. In practice two methods can be used.

A diagram corresponding to Fig. 3 or 4 can be drawn and the appropriate equations written down. There is no need to go into algebra—with the aid of the diagram and common sense they can be written straight away as numbers. This is precisely equivalent to the above use of minus signs, but is less likely to result in mistakes.

Alternatively, the C.G. limits and cockpit C.G.(s) may be converted to the leading edge datum using the measured difference (see para. 4.4), the empty C.G. measured from that datum as usual, and all the calculations made in the standard manner. In this case also a diagram should be drawn, to facilitate making the conversion.

LIST OF SYMBOLS

a — distance from front sling to datum, or from platform load to datum.
E — empty weight.
E^1 — empty weight as weighed, when different from that for the standard condition.
F — front support load.
J — weight of jury ballast.
l — distance from front support to rear support.
P — cockpit load (both sides for side-by-side two-seaters).
P_f — front cockpit load, for tandem two-seaters.
P_r — rear cockpit load, for tandem two-seaters.
R — rear support load.
W — total or all-up-weight.
w_1 etc. weight of missing or surplus equipment items.
x — distance of C.G. from datum point, $+ve$ aft, unless otherwise stated.
$\bar{x}$ — loaded or all-up C.G.
x_a — aft C.G. limit.
x_e — empty C.G.
x_e^1 — empty C.G. as weighed, when different from that for the standard condition.
x_f — forward C.G. limit.
x_p — cockpit load C.G.
x_{pf} — front cockpit load C.G., for tandem two-seaters.
x_{pr} — rear cockpit load C.G., for tandem two-seaters.
x_1 etc. — C.G. of missing or surplus equipment items.

APPENDIX 1

Worked Examples—Empty Condition as Weighed

The following examples all apply to a mythical glider having a given empty weight and C.G. position, with either normal or aft wheel location, taking each of the four support configurations in turn. The datum point is the wing root leading edge.

Method A—Front and Rear Slings—Fig. 1(*a*)

Front sling reading ..	359 lb.
Front sling zero ..	4 lb.
Rear sling reading ..	91 lb.
Rear sling zero ..	1 lb.
Front sling ahead of datum $= a$ ..	0·5 in.
Front sling to rear sling $= l$ ..	133·5 in.

Then $F = 359 - 4 = 355$ lb.

$R = 91 - 1 = 90$ lb.

$\therefore \quad E = 355 + 90 = 445$ lb.

And $x_e = \left(133{\cdot}5 \times \dfrac{90}{445}\right) - 0{\cdot}5$

$= 27{\cdot}0 - 0{\cdot}5 = 26{\cdot}5$ in.

Method B—Platform Scales

(1) Normal wheel location, rear sling—Fig. 1(b)

Platform scale reading	..	404·5 lb.
Platform scale zero	..	2 lb.
Rear sling reading	..	43·5 lb.
Rear sling zero	..	1 lb.
Main wheel aft of datum $= a$	..	15·2 in.
Main wheel to rear sling $= l$	..	118 in.

Then $F = 404{\cdot}5 - 2 \quad = 402{\cdot}5$ lb.

$R = 43{\cdot}5 - 1 \quad = 42{\cdot}5$ lb.

$E = 402{\cdot}5 + 42{\cdot}5 \quad = 445$ lb.

and $x_e = \left(118 \times \frac{42{\cdot}5}{445}\right) + 15{\cdot}2$

$= 11{\cdot}3 + 15{\cdot}2 \quad = 26{\cdot}5$ in.

(2) Aft wheel location, rear sling and jury ballast—Fig. 1(c)

Platform scale reading	..	462 lb.
Platform scale zero	..	2 lb.
Rear sling reading $= R$	..	8 lb.
Rear sling zero, inc. jury ballast $= J$	..	23 lb.
Main wheel aft of datum $= a$	..	30·0 in.
Main wheel to rear sling $= l$	..	103 in.

Then $F = 462 - 2 \quad = 460$ lb.

$J - R = 23 - 8 \quad = 15$ lb.

$E = 460 - 15 \quad = 445$ lb.

and $x_e = 30{\cdot}0 - \left(103 \times \frac{15}{445}\right)$

$= 30{\cdot}0 - 3.5 \quad = 26{\cdot}5$ in.

(3) Aft wheel location, nose support—Fig. 1(d)

Platform scale reading	..	250·5 lb.
Platform scale zero	..	2 lb.
Nose support reading	..	201 lb.
Nose support zero	..	4·5 lb.
Main wheel aft of datum $= a$	..	30·0 in.
Main wheel to nose sling $= l$	..	79·5 in.

Then $F = 201 - 4{\cdot}5 \quad = 196{\cdot}5$ lb.

$R = 250{\cdot}5 - 2 \quad = 248{\cdot}5$ lb.

$E = 196{\cdot}5 + 248{\cdot}5 \quad = 445$ lb.

and $x_e = 30{\cdot}0 - \left(79{\cdot}5 \times \frac{196{\cdot}5}{445}\right)$

$= 30{\cdot}0 - 3{\cdot}5 \quad = 26{\cdot}5$ in.

APPENDIX 2

Worked Example—Empty Weight and C.G. Corrected for Equipment State

The glider of Appendix 1 weighed with 2 lb. of instruments missing from position 30 in. ahead of datum, and parachute weighing 20 lb. in position 6 in. ahead of datum.

$E^1 = 463$ lb.

$x_e^1 = 25{\cdot}35$ in. aft of datum

Then $E = 463 - 2 + 20 \quad = 445$ lb.

$E\,x_e = (463 \times 25{\cdot}35) - (2 \times 30) + (20 \times 6)$

$= 11740 - 60 + 120$

$= 11800$ lb.in.

$x_e = 11800 \div 445 \quad = 26{\cdot}5$ in.

Note that both the correction terms have changed sign in the moments—this is because the x values are negative since the items are both ahead of the C.G.

In practice, the process might be tabulated, thus:

Item	*Weight lb.*	*Arm in.*	*Moment, lb.in. For'd.*	*Aft*
As weighed	463	+25·35		11740
Add instruments	2	—30	60	
Deduct parachute	—20	— 6		120
			60	11860
Empty weight	445	+26·5		11800

APPENDIX 3

Worked Examples—Maximum and Minimum Cockpit Loads

Single-Seater

The glider of Appendix 1 has maximum permissible all-up weight of 670 lb. and forward and aft C.G. limits of 12 in. and 17·2 in. aft of datum respectively. The cockpit C.G. is 12 in. ahead of the datum.

The minimum cockpit load, by equation 5, is

$$445 \times \frac{(26{\cdot}5 - 17{\cdot}2)}{(12 + 17.2)} = 445 \times \frac{9{\cdot}3}{29.2} = 142 \text{ lb.} \quad \ldots \quad 3{\cdot}1$$

The maximum cockpit load, from weight considerations, by equation 6 is

$$670 - 445 = 225 \text{ lb.} \quad \ldots \quad 3{\cdot}2$$

Checking the validity of this by equation 7 yields

$$\bar{x} = \frac{1}{670}(445 \times 26{\cdot}5 - 225 \times 12)$$

$$= \frac{1}{670}(11800 - 2700)$$

$$= 13{\cdot}8 \text{ in.} \quad \ldots \quad 3.3$$

which is within limits. The value 225 lb. is therefore valid.

Alternatively, the maximum cockpit load from C.G. considerations, by equation 8, is:

$$445 \times \frac{(26{\cdot}5 - 12)}{(12 + 12)} = 445 \times \frac{14{\cdot}5}{24} = 268 \text{ lb.} \quad \ldots \quad 3.4$$

and the final maximum cockpit load is the lower of the values at 3.2 and 3.4, viz. 225 lb.

Tandem Two-Seater

A mythical glider of this class, having a swept-forward wing, has the following:

Maximum permissible weight	=	1100 lb.
Empty weight	=	710 lb.
C.G. limits	=	5·2 to 1·0 in. ahead of datum
Empty C.G.	=	10·3 in. aft of datum
Front cockpit C.G.	=	45 in. ahead of datum
Rear cockpit C.G.	=	3 in. ahead of datum

Datum point is wing leading edge at junction with fuselage.

In this example, $x_f = -5{\cdot}2$ and $x_a = -1{\cdot}0$ since both C.G. limits are ahead of the datum, and not aft as in their definitions as positive numbers.

The minimum front cockpit load, by equation 9, is:

$$\frac{710\,(10{\cdot}3 + 1{\cdot}0) - P_r\,(-1{\cdot}0 + 3)}{(45 - 1{\cdot}0)}$$

$$= 182 - 0{\cdot}046\,P_r \quad \ldots \quad 3.5$$

The maximum front cockpit load from C.G. considerations, by equation 11, is:

$$\frac{710\,(10{\cdot}3 + 5{\cdot}2) - P_r\,(-5{\cdot}2 + 3)}{(45 - 5{\cdot}2)}$$

$$= 276 + 0{\cdot}055\,P_r \quad \ldots \quad 3.6$$

The maximum front cockpit load from weight considerations, by equation 10, is:

$1100 - 710 - P_r = 390 - P_r$ 3.7

The tabulation now follows, thus:

1	2	3	4	5	6
P_r lb.	$0{\cdot}046\ P_r$ lb.	Min. P_f lb.	$0{\cdot}055\ P_r$ lb.	Max. P_f lb. C.G.	Weight
0	0	182	0	276	—
100	4	178	6	282	—
120	5	177	7	—	—
140	6	176	8	—	250
160	7	175	9	—	230
180	8	174	10	—	210
200	9	173	11	—	190
220	10	172	12	—	170
240	11	171	13	289	150

In the table:

Col. 1 is the arbitrary values of P_r as in para. 6.2.
Col. 2 is Col. 1 multiplied by 0·046 from equation 3.5.
Col. 3 is 182 from equation 3.5 minus Col. 2.
Col. 4 is Col. 1 multiplied by 0·055 from equation 3.6.
Col. 5 is 276 from equation 3.6 plus Col. 4.
Col. 6 is 390 from equation 3.7 minus Col. 1.

In this example, the maximum front cockpit loads are governed by the 240 lb. overall limit for rear cockpit loads of 140 lb. or less, and by all-up weight for greater rear cockpit loads. For 220 and 240 lb. rear cockpit load, the maximum front cockpit load is less than the minimum. This is obviously invalid, and it means that the rear cockpit load must be limited to the value for which maximum and minimum front cockpit loads are equal; by inspection this is 218 lb.

The limitations card should therefore read:

Rear cockpit load	*Front cockpit load* Max.	Min.
0 lb.	240 lb.	182 lb.
100 lb.	240 lb.	178 lb.
120 lb.	240 lb.	177 lb.
140 lb.	240 lb.	176 lb.
160 lb.	230 lb.	175 lb.
180 lb.	210 lb.	174 lb.
200 lb.	190 lb.	173 lb.
218 lb.	172 lb.	172 lb.

Another mythical glider of this class, having a straight wing, has the following:

Maximum permissible weight .. .. .. 1180 lb.
Empty weight .. .. 780 lb.
C.G. limits .. .. 14 to 18·5 in. aft of datum
Empty C.G. .. .. 32·2 in. aft of datum
Front cockpit C.G. .. .. 40 in. ahead of datum
Rear cockpit C.G. .. .. 5 in. aft of datum
Datum point is wing-root leading edge.

In this example $x_{pr} = -5$ in., since the rear cockpit C.G. is aft of the datum, and not ahead as in the definition as a positive number.

The minimum front cockpit load, by equation 9, is:

$$\frac{780\,(32{\cdot}2 - 18{\cdot}5) - P_r\,(18{\cdot}5 - 5)}{(40 + 18{\cdot}5)}$$

$$= 182 - 0{\cdot}23\,P_r \quad \ldots \quad 3.8$$

The maximum front cockpit load from C.G. considerations, by equation 11, is:

$$\frac{780\,(32{\cdot}2 - 14) - P_r\,(14 - 5)}{(40 + 14)}$$

$$= 263 - 0{\cdot}166\,P_r \quad \ldots \quad 3.9$$

The maximum front cockpit load from weight considerations, by equation 10, is:

$$1180 - 780 - P_r = 400 - P_r \quad \ldots \quad 3.10$$

The tabulation now follows, thus:

1	2	3	4	5	6
P_r lb.	$0{\cdot}23\,P_r$ lb.	*Min.* P_f lb.	$0{\cdot}166\,P_r$ lb.	*Max.* P_f lb. C.G.	*Weight*
0	0	182	0	263	—
100	23	159	17	246	—
120	28	154	20	243	—
140	32	150	23	240	—
160	37	145	27	236	240
180	41	141	30	233	220
200	46	136	33	230	200
220	51	131	37	226	180
240	55	127	40	223	160

Col. 1 is the arbitrary values of P_r as in para. 6.2.
Col. 2 is Col. 1 multiplied by 0·23 from equation 3.8.
Col. 3 is 182 from equation 3.8 minus Col. 2.
Col. 4 is Col. 1 multiplied by 0·134 from equation 3.9.
Col. 5 is 303 from equation 3.9 minus Col. 4.
Col. 6 is 400 from equation 3.10 minus Col. 1.

In this example, the maximum front cockpit loads are governed by the 240 lb. overall limit for rear cockpit loads up to 140 lb., by C.G. for 160 lb. and by weight for 180 lb. or more. The limitations card should therefore read:

Rear cockpit load	*Front cockpit load* *Max.*	*Min.*
0 lb.	240 lb.	182 lb.
100 lb.	240 lb.	159 lb.
120 lb.	240 lb.	154 lb.
140 lb.	240 lb.	150 lb.
160 lb.	236 lb.	145 lb.
180 lb.	220 lb.	141 lb.
200 lb.	200 lb.	136 lb.
220 lb.	180 lb.	131 lb.
240 lb.	160 lb.	127 lb.